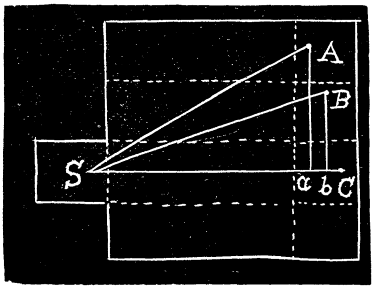


Then $x^2+42x=72$, and $x=1.6495$ feet.

Let $(36+x)^2+(18-x)^2=(42)^2$.

Then $x^2+18x=72$, and $x=3.3693$ feet.

Then if $x<1.6495$ feet, the third route is shortest.

If $x>1.6495$ and <3.3693, the second route is shortest.

If $x>3.3693$, the first route is the shortest.

Since $x=3$, the spider takes the second route, and travels $\sqrt{[(39)^2+(15)^2]}=3\sqrt{[13^2+5^2)}=3\sqrt{(194)}<[3\sqrt{(196)}=3\times14=42]$ feet.

Also solved by C. N. Schmall.

358. Proposed by H. C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Neb.

Cut four coplanar non-copunctual straight lines in a harmonic range.

I. Solution by the PROPOSER.

Let a. b, c, and d be the four lines, not meeting in a point. Let a and b meet at x. Draw two lines cutting a, b, and d at A, A_1, A_2 and A, B_1, B_2, respectively. Draw A_1B_2 and A_2B_1 meeting at C_1. Draw AC_1 cutting b and d at C and C_2. Draw XC_2, and draw XC_1 cutting C at D_2. Draw C_2D_2 cutting a and b at D and D_1. $DD_1D_2C_2$ is the required range.

For ACC_1C_2 is a harmonic range determined by the four-point $A_1B_1B_2A_2$. Hence, $X-ACC_1C_2$ is a harmonic pencil. So also is $X-DD_1D_2C$; therefore DD_1D_2C is the required harmonic range.

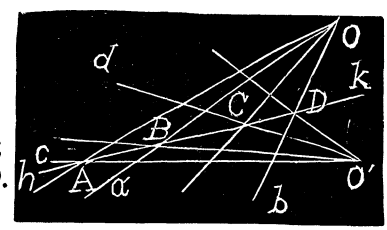
Also solved by C. N. Schmall.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let Oa, Ob, $O'c$, $O'd$ be the four lines; hk the transversal intersecting $O'c$, Oa, $O'd$, Ob in A, B, C, D, respectively.

Let $\alpha=0$ be the equation to Oa; $\beta=0$ the equation to Ob; $\gamma=0$ the equation to $O'c$; $l\alpha-m\beta+n\gamma=0$, the equation to $O'd$.

Then for a harmonic range,



$\alpha-p\beta=0$ is the equation to OC;
$\alpha+p\beta=0$ is the equation to OA;
$\gamma-q(l\alpha-m\beta+n\gamma)=0$ is the equation to $O'B$;
$\gamma+q(l\alpha-m\beta+n\gamma)=0$ is the equation to $O'D$.

$\left\{-\dfrac{2p\Delta}{b-ap}, \dfrac{2\Delta}{b-ap}, 0\right\}$, are the coordinates of A;

$\left\{\dfrac{2np\triangle}{apm+bn-clp+cm}, \dfrac{2n\triangle}{apm+bn-clp+cm}, -\dfrac{2\triangle(lp-m)}{apm+bn-clp+cm}\right\}$ are the coordinates of C;

$\left\{0, \dfrac{2\triangle(qn-1)}{b(qn-1)+cmq}, \dfrac{2\triangle mq}{b(qn-1)+cmq}\right\}$ are the coordinates of B;

$\left\{\dfrac{2\triangle(qn+1)}{a(qn+1)-qcl}, 0, -\dfrac{2\triangle ql}{a(qn+1)-qcl}\right\}$ are the coordinates of D.

$\alpha mq+\beta pmq-\gamma p(qn-1)=0$, is the line through AB;
$\alpha nql-\beta(qmn+m-lp)+\gamma n(qn+1)=0$, is the line through CD.

Comparing, we get $q=0$ or $q=-(1/n)$, $p=m/l$ or $p=0$. There are no positive values for q when p is positive. When p is positive, $q=0$, etc.

Whatever relations we establish we cannot find p and q both real or both positive.

CALCULUS.

286. Proposed by R. D. CARMICHAEL, Princeton University.

Solve the differential equation

$$[a_0x^3+a_1x^2y+a_2xy^2+(a_0-a_1+a_2)y^3 \\ +a_3x^2+a_4xy+a_5y^2+a_6x+a_7y+a_8]dx \\ +[a_0y^3+a_1xy^2+a_2x^2y+(a_0-a_1+a_2)x^3 \\ +a_3y^2+a_4xy+a_5x^2+a_6y+a_7x+a_8]dy=0.$$

No solution of this problem has been received.

287. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

An object P, being placed beyond the principal focus F of a convex lense, determine its position when its distance PQ, from its image Q, is a minimum.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let u=distance of object from lense, v=distance of image from lense, t=thickness of lense, and r, s=the radii of the first and second surface.

Then $\dfrac{1}{\dfrac{1}{v}+\dfrac{\mu-1}{s}}+\dfrac{1}{\dfrac{1}{u}-\dfrac{\mu-1}{r}}=\dfrac{t}{\mu}$, where μ=index of refraction, or

$$\mu\left(\frac{1}{u}+\frac{1}{v}\right)=\left(\frac{1}{r}-\frac{1}{s}\right)\mu(\mu-1)+t\left(\frac{1}{v}+\frac{\mu-1}{s}\right)\left(\frac{1}{u}-\frac{\mu-1}{r}\right)\ldots(1).$$